ELECTRODE LAYERS IN FLOWS OF AN INVISCID PLASMA WITH GOOD CONDUCTIVITY

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The structure of the electromagnetic electrode layers that are produced in flows across a magnetic field by a completely ionized and inviscid plasma with good conductivity and a high magnetic Reynolds number is examined in a linear approximation. Flow past a corrugated wall and flow in a plane channel of slowly varying cross section with segmented electrodes are taken as specific examples. The possibility is demonstrated of the formation of "nondissipative" electrode layers with thicknesses on the order of the Debye distance or electron Larmor radius and of "dissipative" layers with thicknesses on the order of the skin thickness, as calculated from the diffusion rate in a magnetic field [2].

In plasma flow in a transverse magnetic field, near the walls, along with the "gasdynamic" boundary layers, which owe their formation to viscosity, thermal conductivity, etc. (because of the presence of electromagnetic fields, their structures may vary considerably from that of ordinary gasdynamic layers), proper electromagnetic boundary layers may also be produced. An example of such layers is the Debye layer in which the quasi-neutrality of the plasma is upset. No less important, in a number of cases, is the quasi-neutral electromagnetic boundary layer, in which there is an abrupt change in the "frozen-in" parameter $k = B/\rho$ (B is the magnetic field and ρ is the density of the medium). This layer plays a special role when we must explicitly allow for the Hall effect and the related formation of a longitudinal electric field (in the direction of the velocity v of the medium). We will call this the magnetic layer. The magnetic boundary layer can be "dissipative" as well as "nondissipative" (see below). The "dissipative" magnetic layer has been examined in a number of papers: for an incompressible medium with a given motion law in [1], for a compressible medium with good conductivity in [2], and with poor conductivity in [3]. In the present paper, particular attention will be devoted to nondissipative magnetic boundary layers.

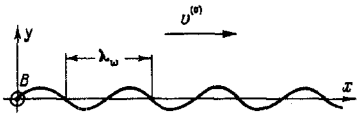

Fig. 1

The nature of electromagnetic and, in particular, magnetic boundary layers in flow in channels across a magnetic field is greatly affected by two factors: the conductivity of the plasma and the degree of manifestation of the Hall effect, which is characterized by the transfer parameter

$$\xi = I / envf ,$$

where f is the width, I is the discharge current, v is the velocity of the plasma, n is the number of particles per cm^3, and e the elementary charge (see [4]).

In addition, the plasma flow is characterized by the magnetic Reynolds number

$$R_m = vL / \nu_m \qquad (\nu_m = c^2 / 4\pi\sigma).$$

Here, L is the characteristic longitudinal length scale, ν_m is the magnetic viscosity of the medium, and σ is the conductivity of the medium. If ξ is small (the trajectories of the ion and electron components of the plasma virtually coincide) and the condition

$$R_m \gg L^2 / f^2$$

is satisfied, the electromagnetic layers will be localized near the walls. If the effect of the finite conductivity of the plasma (i.e., dissipation) must be taken into account here, "dissipative" layers are formed whose thickness increases according to the diffusion law and is a function of the conductivity. But if the conductivity of the plasma can be considered infinite, "nondissipative" layers are formed whose thickness is determined by the local characteristics of the stream and is on the order of the "local" Debye distance or electron Larmor radius.

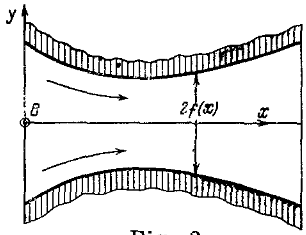

Fig. 2

If, however, $\xi \gtrsim 1$ (the trajectories of the ion and electron components differ greatly; more precisely—the angle between them—is on the order of f/L), the perturbations of the "frozen-in" parameter k and of other plasma characteristics will be transferred by electrons throughout the entire volume of the channel, which, with allowance for dissipation, leads to the formation of a single anode layer [2].

Owing to this "transfer of perturbations by electrons," when $\xi \gtrsim 1$, "poor" boundary conditions can completely disrupt the "ideal" flow pattern.

Below we shall be concerned only with electromagnetic layers, and, with this in mind, we shall consider the two-dimensional flow across a magnetic field of a completely ionized and inviscid plasma with good conductivity and a high magnetic Reynolds number $R_m \gg L^2/f^2$, under the assumption that $L^2/f^2 \gg 1$. The case of $R_m \ll 1$ has been considered many times.

Even the calculation of ordinary gasdynamic boundary layers is, as a rule, a very complicated problem. But in the case of plasma flow in an electromagnetic field, the situation becomes even more difficult, since the number of electromagnetic characteristics and parameters is greater. In gasdynamics, however, qualitatively correct results are obtained for linearized problems, when flow perturbations caused by the boundary conditions are considered small. Therefore, we shall consider only a linear approximation for perturbations. This is all the more justified for flows with $\xi \gtrsim 1$, since it is clear from the above that such a flow cannot be divided into a main stream and boundary layers and, therefore, correct nonlinear calculation by the boundary-layer approximation method is meaningless.

§1. Initial equation system. Problem of flow of a plasma past a corrugated wall. Steady plasma flows are described by the equation system of the two-fluid hydrodynamic approximation (for the ion and electron components) and by the Maxwell equations

$$M(\mathbf{v}_i\nabla)\mathbf{v}_i = -\frac{\nabla p_i}{n_i} + e\left(\mathbf{E} + \frac{\mathbf{v}_i}{c}\times\mathbf{B}\right) - m\frac{n_e}{n_i}\frac{\mathbf{v}_i - \mathbf{v}_e}{\tau},$$

$$m(\mathbf{v}_e\nabla)\mathbf{v}_e = -\frac{\nabla p_e}{n_e} - e\left(\mathbf{E} + \frac{\mathbf{v}_e}{c}\times\mathbf{B}\right) + m\frac{\mathbf{v}_i - \mathbf{v}_e}{\tau},$$

$$\operatorname{div} n_i \mathbf{v}_i = 0, \quad \operatorname{div} n_e \mathbf{v}_e = 0, \quad \operatorname{div} \mathbf{B} = 0, \quad \operatorname{rot} \mathbf{E} = 0,$$
$$\operatorname{div} \mathbf{E} = 4\pi e (n_i - n_e), \quad \sigma = n_e e^2 \tau / m,$$
$$\operatorname{rot} \mathbf{B} = 4\pi c^{-1} e (n_i \mathbf{v}_i - n_e \mathbf{v}_e),$$
$$p_i = p_i(n_i), \quad p_e = p_e(n_e), \tag{1.1}$$

where M and m are the ion (assumed to be singly charged) and electron masses; $n_{i,e}$, $p_{i,e}$ and $v_{i,e}$ are the number of particles per cm^3, the gas-kinetic pressure, and the velocity of the components, respectively; E and B are the electric and magnetic vectors; τ is the electron-ion collision time; and σ is the conductivity of the plasma.

First, let us consider flow with a small transfer parameter ξ, and namely, flow of a homogeneous plasma stream past a corrugated wall (Fig.1). Let the velocity $\mathbf{v}^{(0)}$ of the unperturbed (by the wall) flow be directed along the x axis. The magnetic field B is perpendicular to the plane of the figure. For the linear theory to be applicable, and this will be assumed below, it is necessary that the height of the wall projections a be small in comparison with the distance λ_w between them (light corrugation)

$$a / \lambda_w \ll 1. \tag{1.2}$$

In the given problem, ξ is on the order of a/λ_w.

In accordance with this, perturbations $F^{(1)}$ of the flow parameters (density, velocity, etc.) are considered small in comparison with the unperturbed values of $F^{(0)}$

$$F^{(1)} \sim (a / \lambda_w) F^{(0)} \ll F^{(0)}. \tag{1.3}$$

If we give the perturbations in the form exp (ikr), where $\mathbf{k} = \{q, \varkappa\}$, $q = 2\pi/\lambda_w$, after linearization we obtain from (1.1)

$$iMqv^{(0)}\mathbf{v}_i^{(1)} = -i\mathbf{k}c_i^2 \frac{n_i^{(1)}}{n^{(0)}} + e\left(\mathbf{E}^{(1)} + \frac{\mathbf{v}_i^{(1)}}{c} \times \mathbf{B}^{(0)} + \frac{\mathbf{v}^{(0)}}{c} \times \mathbf{B}^{(1)}\right) - \frac{m}{\tau}(\mathbf{v}_i^{(1)} - \mathbf{v}_e^{(1)}),$$
$$imqv^{(0)}\mathbf{v}_e^{(1)} = -i\mathbf{k}c_e^2 \frac{n_e^{(1)}}{n^{(0)}} - e\left(\mathbf{E}^{(1)} + \frac{\mathbf{v}_e^{(1)}}{c} \times \mathbf{B}^{(0)} + \frac{\mathbf{v}^{(0)}}{c} \times \mathbf{B}^{(1)}\right) + \frac{m}{\tau}(\mathbf{v}_i^{(1)} - \mathbf{v}_e^{(1)}),$$
$$n^{(0)}(\mathbf{k} \cdot \mathbf{v}_i^{(1)}) + qv^{(0)} n_i^{(1)} = 0,$$
$$n^{(0)}(\mathbf{k} \cdot \mathbf{v}_e^{(1)}) + qv^{(0)} n_e^{(1)} = 0$$
$$\mathbf{k} \cdot \mathbf{B}^{(1)} = 0, \quad \mathbf{k} \times \mathbf{B}^{(1)} = -\frac{4\pi i e}{c}\{n^{(0)}(\mathbf{v}_i^{(1)} - \mathbf{v}_e^{(1)}) + \mathbf{v}^{(0)}(n_i^{(1)} - n_e^{(1)})\},$$
$$\mathbf{k} \times \mathbf{E}^{(1)} = 0, \quad \mathbf{k} \cdot \mathbf{E}^{(1)} = -4\pi i e (n_i^{(1)} - n_e^{(1)}),$$
$$c_i^2 = \frac{dp_i^{(0)}}{M\, dn^{(0)}}, \quad c_e^2 = \frac{dp_e^{(0)}}{m\, dn^{(0)}}, \tag{1.4}$$

where c_i and c_e are the thermal "acoustic" velocities of the components, and the superscripts 0 and 1 denote unperturbed and perturbed values, respectively.

We shall also assume that the vectors $\mathbf{v}_{i,e}^{(1)}$ and $\mathbf{E}^{(1)}$ lie in the xy plane, and that $\mathbf{B}^{(1)}$ has only a transverse (along the z axis) component. For system (1.4) to have nontrivial solutions, its determinant must vanish, and the components of the vector k are thus linked by the so-called dispersion equation.

The unsteady perturbations of the type exp $(i\mathbf{k} \cdot \mathbf{r} - i\omega t)$ for a medium at rest are treated in the conventional theory of plasma oscillations, where the dispersion equation is

$$D(\omega, \mathbf{k}) = 0. \tag{1.5}$$

The dependence of ω on the components of vector k is found from this.

When $\omega = -\mathbf{k} \cdot \mathbf{v}^{(0)}$, the dispersion equation has the form

$$D(\mathbf{k} \cdot \mathbf{v}^{(0)}, \mathbf{k}) = 0, \tag{1.6}$$

thereby giving the relationship between the components of the wave vector. In our specific case,

$$\varkappa = \varphi_s(q) \qquad (s = 1, 2, \ldots). \tag{1.7}$$

Generally speaking, we see that Eqs. (1.5) and (1.6) differ from one another.

In view of the awkwardness of the general dispersion equation of system (1.4), we shall consider the cases of "good" and "poor" conductivity separately.

First of all, we assume that $qv^{(0)}\tau \gg 1$. This means that the probability of an electron-ion collision during passage by a wall projection is small. The conductivity of the plasma can then be considered infinite and the dispersion equation has the form

$$\left[k^2c_e^2 + \omega_{pe}^2 + \omega_e^2 \frac{\omega_{pi}^2 + k^2c^2}{\omega_{pi}^2 + \omega_{pe}^2 + k^2c^2} - q^2 v_{(0)}^2\right] \times \left[k^2c_e^2 + \omega_{pi}^2 + \omega_i^2 \frac{\omega_{pe}^2 + k^2c^2}{\omega_{pi}^2 + \omega_{pe}^2 + k^2c^2} - q^2 v_{(0)}^2\right] =$$
$$= \left[\omega_{pi}^2 + \omega_i^2 \frac{\omega_{pe}^2}{\omega_{pi}^2 + \omega_{pe}^2 + k^2c^2}\right] \times \left[\omega_{pe}^2 + \omega_e^2 \frac{\omega_{pi}^2}{\omega_{pi}^2 + \omega_{pe}^2 + k^2c^2}\right],$$
$$k^2 = q^2 + \varkappa^2, \quad \omega_{pi}^2 = \frac{4\pi n^{(0)} e^2}{M},$$
$$\omega_{pe}^2 = \frac{4\pi n^{(0)} e^2}{m}, \quad \omega_i = \frac{eB^{(0)}}{Mc}, \quad \omega_e = \frac{eB^{(0)}}{mc}. \tag{1.8}$$

If

$$q^2 \Lambda_i^2 \ll \frac{M}{m} \qquad \left(\Lambda_i = \frac{Mcv^{(0)}}{eB^{(0)}}\right)$$

where Λ_i is the ion Larmor radius determined from the velocity $v^{(0)}$, we obtain

$$\varkappa_1^2 = q^2 \frac{v_{(0)}^2 - c_H^2}{c_H^2}, \quad \varkappa_2^2 = -\frac{\omega_{pe}^2}{c^2}\frac{c_H^2}{c_T^2},$$
$$\varkappa_3^2 = -\frac{\omega_{pi}^2}{c_i^2} - \frac{\omega_{pe}^2}{c_e^2} = -\left(\frac{1}{D_i^2} + \frac{1}{D_e^2}\right),$$
$$c_H^2 = c_T^2 + c_A^2, \quad c_T^2 = c_i^2 + \frac{m}{M} c_e^2,$$
$$c_A^2 = \frac{B_{(0)}^2}{4\pi M n^{(0)}}. \tag{1.9}$$

Here, c_T is the thermal speed of sound, c_A the Alfven velocity, and c_H the magnetic speed of sound.

But if $\Lambda_i^2 q^2 \gg M/m$,

$$\varkappa_1^2 = -q^2\left(1 - \frac{v_{(0)}^2}{c_T^2}\right), \quad \varkappa_2^2 = -q^2,$$

$$\varkappa_3^2 = -\frac{\omega_{pe}^2}{c_e^2} - \frac{\omega_{pi}^2}{c_i^2 - v_{(0)}^2} - q^2. \tag{1.10}$$

In both limiting cases, the root $\varkappa_1^2$ describes magnetosonic perturbations which penetrate deep into the stream in supersonic ($v^{(0)} > c_\varkappa$) flows. The other two roots give surface waves, i. e., the corresponding perturbations are localized near the wall. One of them ($\varkappa_3^2$ arises when quasi-neutrality is upset and, as is easy to see, is related to the Debye shielding distance. The root $\varkappa_2^2$ in the first limiting case equals $-\omega_{pe}^2/c_T^2$ when $c_T \gg c_A$ (transverse electron oscillations) or $-\omega_e^2/c_T^2$ when $c_T \gg c_A$ (in this case, the layer thickness $\delta_2 = 1/|\varkappa_2|$ equals the electron Larmor radius, as calculated from the velocity c_T). In the second limiting case, the root $\varkappa_2^2$ describes harmonic (vacuum) perturbations. The roots $\varkappa_{2,3}$ obviously describe "nondissipative" boundary layers whose thickness is determined by local stream characteristics, i. e., it does not increase in space, as occurs, for example, in ordinary viscous boundary layers. The presence of the three roots $\varkappa$ allows us to set three boundary conditions at the wall. These might be a condition on the velocity component normal to the wall, a condition on the electric field, and a condition on the electric current.

Let us consider the case $qv_{(0)}\tau \ll 1$, when the effect of finite conductivity must be considered. For a quasi-neutral plasma we obtain the following dispersion equation:

$$(q^2v_{(0)}^2 - k^2c_T^2)\frac{c^2k^2}{\omega_{pe}^2\tau} = iqv_{(0)}(q^2v_{(0)}^2 - k^2c_\varkappa^2). \tag{1.11}$$

If $(qc^2 / v_{(0)}\omega_{pe}^2\tau) \to 0$,

$$\varkappa_1^2 = q^2\frac{v_{(0)}^2 - c_\varkappa^2}{c_\varkappa^2}, \qquad \varkappa_2^2 = iqv_{(0)}\frac{c_\varkappa^2}{\nu_m c_T^2}$$

$$\left(\nu_m = \frac{c^2}{4\pi\sigma}\right). \tag{1.12}$$

As before, the first root describes the magnetosonic perturbations and the second root gives the "dissipative" magnetic boundary layer. The thickness $\delta_2 = 1/|\varkappa_2|$ of this layer equals the skin thickness, as calculated from the diffusion rate in the field. This means that under real conditions the thickness of this layer will increase with distance from the leading edge.

§2. Nondissipative flow in a channel of slowly varying cross section for an arbitrary transfer parameter. The steady two-dimensional flow of a plasma across a magnetic field in a channel of slowly varying cross section (see Fig. 2), i. e., when the length L of the channel is small as compared with its width $2f$, will be examined below. The flow will be considered nondissipative; dissipative flows of a quasi-neutral plasma were investigated earlier [2]. The electrode-walls of the channel are assumed to be in narrow (more precisely, infinitely narrow) segments that are electrically insulated from one another. As is known, electrode segmenting makes it possible in principle to accomplish, in a narrow channel, quasi-one-dimensional flow in which all of the flow parameters (except the potential) are slow functions only of the longitudinal coordinate x.

While it is easy to short circuit the segments by placing high but finite resistances between them, small perturbations of the ideal boundary conditions arise and, as a result, boundary layers are produced.

In the absence of dissipation, the flows are conveniently described by the stream functions $\psi_{i,e}$, which are introduced as follows:

$$n_k v_{kx} = \frac{\partial\psi_k}{\partial y}, \quad n_k v_{ky} = -\frac{\partial\psi_k}{\partial x} \quad (k = i, e). \tag{2.1}$$

In the above two-fluid model, the system of equations describing the flow have the form [5]

$$\frac{M}{2n_i^2}\left[\left(\frac{\partial\psi_i}{\partial x}\right)^2 + \left(\frac{\partial\psi_i}{\partial y}\right)^2\right] + w_i + e\varphi = U_i(\psi_i),$$

$$\frac{m}{2n_e^2}\left[\left(\frac{\partial\psi_e}{\partial x}\right)^2 + \left(\frac{\partial\psi_e}{\partial y}\right)^2\right] + w_e - e\varphi = U_e(\psi_e),$$

$$w_k = \int\frac{1}{n_k}dp_k,$$

$$\frac{B}{n_i} + \frac{c}{e}\frac{dU_i}{d\psi_i} = \frac{Mc}{en_i}\left[\frac{\partial}{\partial x}\left(\frac{1}{n_i}\frac{\partial\psi_i}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{1}{n_i}\frac{\partial\psi_i}{\partial y}\right)\right],$$

$$B = \frac{4\pi}{c}e(\psi_i - \psi_e).$$

$$\frac{B}{n_e} - \frac{c}{e}\frac{dU_e}{d\psi_e} = -\frac{mc}{en_e}\left[\frac{\partial}{\partial x}\left(\frac{1}{n_e}\frac{\partial\psi_e}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{1}{n_e}\frac{\partial\psi_e}{\partial y}\right)\right],$$

$$\Delta\varphi = -4\pi e(n_i - n_e). \tag{2.2}$$

Here, $U_{i,e}$ are arbitrary functions of ψ_i and ψ_e respectively, and w_k denotes the "enthalpies" of the components, while φ is the electrostatic potential. For a channel of slowly varying cross section, system (2.2) can be significantly simplified with a boundary-layer approximation, i. e., neglecting the square of the first derivatives and neglecting the second derivatives with respect to the longitudinal coordinate x, since they are small, on the order of f^2/L^2, as compared to the derivatives with respect to the transverse coordinate y [5].

We take the quasi-one-dimensional flow as the unperturbed flow (whose characteristics have the superscript 0):

$$n_i^{(0)} = n_e^{(0)} = n_0(x), \quad B^{(0)} = B_0(x),$$

$$U_i^{(0)} = U_{0i} - eB_0c^{-1}n_0^{-1}\psi_i^{(0)},$$

$$U_e^{(0)} = U_{0e} + eB_0c^{-1}n_0^{-1}\psi_e^{(0)}, \quad U_{0ie} = \text{const},$$

$$\psi_i^{(0)} = (n_0v_0f)\zeta, \quad \psi_e^{(0)} = (n_0v_0f)(\zeta - \xi b_0),$$

$$\zeta = y/f, \quad \xi = cB_0(0)/4\pi e(n_0v_0f),$$

$$b = B/B_0(0), \quad B_0/n_0 = \text{const}, \quad n_0v_0f = \text{const},$$

$$\varphi^{(0)} = \varphi_{00}(x) - c^{-1}n_0^{-1}B_0(n_0v_0f)\zeta,$$

$$Mv_0^2/2 + w_i^{(0)}(n_0) + e\varphi_{00} = U_{0i} = \text{const}. \tag{2.3}$$

Here, f is the channel half-width; ξ is the transfer parameter for a channel of infinite length, where $B_0 \to 0$ as $x \to L$; and the coordinate origin $x = 0$ is set at the channel inlet, where $v_0 = 0$.

We introduce the dimensionless values

$$\Psi = \frac{\psi}{n_0v_0f}, \quad \Phi = \frac{e\varphi}{Mc_A^2(0)},$$

$$N = \frac{n}{n_0} \quad \left(c_A^2(0) = \frac{B_0^2(0)}{4\pi Mn_0(0)}\right). \tag{2.4}$$

Assuming that the perturbations of the quasi-one-dimensional flow are small, let us linearize system (2.2), omitting the perturbation superscript 1,

$$\xi b = \Psi_i - \Psi_e, \quad \frac{c_A^2(0)}{(f\omega_p)^2}\frac{\partial^2\Phi}{\partial\zeta^2} =$$

$$= N_e - N_i\left(\omega_p^2 = \frac{4\pi e^2n_0}{M}\right), \quad \left(\frac{\xi v_0^2}{c_A^2(0)}\frac{\partial}{\partial\zeta} + 1\right)\Psi_i +$$

$$+\frac{\xi c_i^2}{c_A^2(0)}\left(1-\frac{v_0^2}{c_i^2}\right)N_i+\xi\Phi=g_i(\zeta)\,,$$

$$\left(\frac{m}{M}\frac{\xi v_0^2}{c_A^2(0)}\frac{\partial}{\partial\zeta}-1\right)\Psi_e+$$

$$+\frac{\xi c_e^2}{c_A^2(0)}\left(1-\frac{m}{M}\frac{v_0^2}{c_e^2}\right)N_e-\xi\Phi=g_e(\zeta-\xi b_0)\,,$$

$$\Psi_i-\Psi_e-\xi b_0N_i+\xi b_0\frac{dg_i(\zeta)}{d\zeta}=$$

$$=\frac{\xi^2v_0^2b_0}{c_A^2(0)}\frac{\partial}{\partial\zeta}\left(\frac{\partial\Psi_i}{\partial\zeta}-N_i\right),$$

$$\Psi_i-\Psi_e-\xi b_0N_e-\xi b_0\frac{\partial g_e(\zeta-\xi b_0)}{\partial\zeta}=$$

$$=-\frac{m}{M}\frac{\xi^2v_0^2b_0}{c_A^2(0)}\frac{\partial}{\partial\zeta}\left(\frac{\partial\Psi_e}{\partial\zeta}-N_e\right),$$

$$g_i(\zeta)=\frac{\xi}{Mc_A^2(0)}\delta U_i(\zeta),$$

$$g_e(\zeta-\xi b_0)=\frac{\xi}{Mc_A^2(0)}\delta U_e(\zeta-\xi b_0)\,. \qquad (2.5)$$

It was assumed in the linearization that the perturbations of the function $U_k(\psi_k)$ had the form

$$U_k^{(1)}=\delta U_k(\psi_k^{(0)})-\frac{e_kB_0}{cn_0}\psi_k^{(1)} \qquad (k=i,\,e)\,. \qquad (2.6)$$

It is easy to see that the functions δU_k correspond to the presence of isomagnetic perturbations

$$\delta\left(\frac{B}{n_k}\right)=\frac{B_0}{n_0}\left(\frac{B^{(1)}}{B_0}-\frac{n_k^{(1)}}{n_0}\right),$$

at the inlet to the channel and on the walls. The velocities c_k are introduced from the relations

$$dw_k=Mc_k^2dN_k\,,$$

and are the speeds of sound for ions and electrons. Instead of Eqs. (2.5.5) and (2.5.6), we can, using (2.5.3) and (2.5.4), obtain

$$\Psi_e-\Psi_i=\frac{c_A^2(0)}{(j\omega_p)^2}\frac{\partial\Phi}{\partial\zeta}+u(x)+$$

$$+\xi\left(\frac{c_i^2}{c_A^2(0)}N_i+\frac{c_e^2}{c_A^2(0)}N_e\right), \qquad (2.7)$$

$$\Psi_e-\Psi_i=\xi b_0\left[\frac{\partial}{\partial\zeta}\left(\Psi_i+\right.\right.$$

$$\left.\left.+\xi\Phi+\xi\frac{c_i^2}{c_A^2(0)}N_i\right)-N_i\right]. \qquad (2.8)$$

Here, u(x) is an arbitrary function. The system of equations (2.5), (2.7), and (2.8) reduces to the following two equations

$$\alpha_i\frac{\partial\Psi_i}{\partial\zeta}+\beta_iN_i=a_i,\qquad \alpha_e\frac{\partial\Psi_i}{\partial\zeta}+\beta_eN_i=a_e\,, \qquad (2.9)$$

$$\alpha_i=\frac{1}{\xi}\left(\frac{c_A(0)}{j\omega_p}\right)^2+\left[\frac{c_e^2+v_0^2}{(j\omega_p)^2}-\right.$$

$$\left.-b_0\left(\frac{\xi v_0}{c_A(0)}\right)^2\right]\frac{\partial}{\partial\zeta}+\xi\left(\frac{v_0c_e}{j\omega_pc_A(0)}\right)^2\frac{\partial^2}{\partial\zeta^2}\,,$$

$$\alpha_e=\xi\left(\frac{v_0}{c_A(0)}\right)^2+\left(\frac{c_A(0)}{j\omega_p}\right)^2+$$

$$+\left(\frac{v_0}{j\omega_p}\right)^2\frac{\partial}{\partial\zeta}-\frac{m}{M}\xi\left(\frac{c_ev_0}{c_A(0)j\omega_p}\right)^2\frac{\partial^2}{\partial\zeta^2}-$$

$$-\frac{m}{M}\left(\frac{\xi v_0^2c_e}{j\omega_pc_A^2(0)}\right)^2\frac{\partial^3}{\partial\zeta^3}\,,$$

$$\beta_i=-\xi\left(\frac{c_{\text{н}}}{c_A(0)}\right)^2+\left[\frac{c_i^2-v_0^2}{(j\omega_p)^2}+\right.$$

$$\left.+b_0\left(\frac{\xi v_0}{c_A(0)}\right)^2\right]\frac{\partial}{\partial\zeta}+\xi(c_i^2-v_0^2)\left(\frac{c_e}{c_A(0)j\omega_p}\right)^2\frac{\partial^2}{\partial\zeta^2}\,,$$

$$\beta_e=-\xi\left(\frac{v_0}{c_A(0)}\right)^2+\left[\frac{m}{M}\left(\frac{\xi v_0c_T}{c_A^2(0)}\right)^2+\frac{c_i^2-v_0^2}{(j\omega_p)^2}\right]\frac{\partial}{\partial\zeta}+$$

$$+\frac{m}{M}\xi(v_0^2-c_i^2)\left(\frac{c_ev_0}{j\omega_pc_A^2(0)}\right)^2\frac{\partial^3}{\partial\zeta^3}\,,$$

$$a_i=u(x)+\left[\frac{1}{\xi}\left(\frac{c_A(0)}{j\omega_p}\right)^2-\right.$$

$$\left.-\xi b_0+\left(\frac{c_e}{j\omega_p}\right)^2\frac{\partial}{\partial\zeta}\right]\frac{\partial}{\partial\zeta}g_i(\zeta)\,,$$

$$a_e=u(x)+g_e(\zeta-\xi b_0)+$$

$$+\left\{\left[1-\frac{m}{M}\xi\left(\frac{v_0}{c_A(0)}\right)^2\frac{\partial}{\partial\zeta}\right]\left[\frac{1}{\xi}\left(\frac{c_A(0)}{j\omega_p}\right)^2+\right.\right.$$

$$\left.+\left(\frac{c_e}{j\omega_p}\right)^2\frac{\partial}{\partial\zeta}\right]\frac{\partial}{\partial\zeta}+$$

$$\left.+\left(\frac{c_e}{j\omega_p}\right)^2\left[\frac{m}{M}\left(\frac{v_0}{c_e}\right)^2-1\right]\frac{\partial^2}{\partial\zeta^2}+1\right\}g_i(\zeta)$$

$$(c_T^2=c_i^2+c_e^2,\quad c_{\text{н}}^2=c_T^2+c_A^2)\,. \qquad (2.10)$$

The operators α and β are commutative. From system (2.9) we obtain

$$(\alpha_e\beta_i-\alpha_i\beta_e)N_i=\alpha_ea_i-\alpha_ia_e,$$

$$(\alpha_e\beta_i-\alpha_i\beta_e)\frac{\partial}{\partial\zeta}\Psi_i=\beta_ia_e-\beta_ea_i\,. \qquad (2.11)$$

Let $g_i(\zeta)=0$, i.e., perturbations $(b-N_i)$ are absent at the inlet. In this case,

$$\frac{\partial}{\partial\zeta}\left(\alpha_i\frac{\partial\Psi_i}{\partial\zeta}+\beta_iN_i\right)=0,$$

$$\frac{\partial}{\partial\zeta}\left(\alpha_e\frac{\partial\Psi_i}{\partial\zeta}+\beta_eN_i\right)=\frac{\partial}{\partial\zeta}g_e(\zeta-\xi b_0)\,. \qquad (2.12)$$

Let us introduce the function z, assuming that $\partial^2\Psi_i/\partial\zeta^2=-\beta_iz,\partial N_i/\partial\zeta=\alpha_iz$. Then the first equation will be satisfied and the second will yield

$$(\alpha_i\beta_e-\alpha_e\beta_T)z=\frac{\partial}{\partial\zeta}g_e(\zeta-\xi b_0)\,, \qquad (2.13)$$

or

$$\left(\frac{\partial}{\partial x}+\xi\frac{db_0}{dx}\frac{\partial}{\partial\zeta}\right)(\alpha_i\beta_e-\alpha_e\beta_i)z=0\,,$$

$$\alpha_i\beta_e-\alpha_e\beta_i=$$

$$=\frac{m}{M}\left(\frac{c_ec_ic^2}{c_Ac_A(0)(j\omega_p)^2}\right)^2\left[\frac{\partial^4}{\partial\zeta^4}-\frac{M}{m}\frac{c_i^2-v_0^2}{\xi c_i^2c^2}\frac{\partial^3}{\partial\zeta^3}-\right.$$

$$-\left(\frac{c_Tj\omega_p}{c_ic_e}\right)^2\frac{\partial^2}{\partial\zeta^2}+$$

$$\left.+\frac{M}{m\xi}\left(\frac{c_{\text{н}}c_A(0)j\omega_p}{cc_ec_i}\right)^2\frac{\partial}{\partial\zeta}+\frac{M}{m}\left(\frac{c_{\text{н}}(j\omega_p)^2}{cc_ec_i}\right)^2\right]. \qquad (2.14)$$

The solutions of the equation $(\alpha_i\beta_e - \alpha_e\beta_i) F = 0$ will be

$$F_1 = e^{\pm\varkappa_1\zeta}, \quad \varkappa_1^2 = \left(\frac{f\omega_p c_T}{c_i c_e}\right)^2, \quad F_2 = e^{\pm\varkappa_2\zeta},$$

$$\varkappa_2^2 = \left(\frac{f\omega_p c_H}{c c_T}\sqrt{\frac{M}{m}}\right)^2. \qquad (2.15)$$

Direct substitution of $\varkappa_{1,2}$ into (2.14) makes it clear that the terms containing the first and third derivatives with respect to ζ are small. The root $\varkappa_1$ corresponds to an electromagnetic layer whose thickness is on the order of the Debye shielding distance, and the root $\varkappa_2$ corresponds to a layer with a thickness on the order of the electron Larmor radius (when $c_A \gg c_T$), as calculated from the "total" speed of sound c_T. When $\xi \to \infty$, the solution of Eq. (2.14) has the form

$$z = \chi(\zeta - \xi b_0) + f_{11}(x) \exp[\varkappa_1(\zeta - 1)] + f_{12}(x) \exp[-\varkappa_1(\zeta + 1)] + f_{13}(x) \exp[\varkappa_2(\zeta - 1)] + f_{14}(x) \exp[-\varkappa_2(\zeta + 1)]. \qquad (2.16)$$

This corresponds to the presence of electrode layers and to the transfer of perturbations along the unperturbed electron trajectories. If $g_i(\zeta) \neq 0$, we must solve Eqs. (2.11) and (2.12), which are more complicated than (2.14). At finite ξ, Eq. (2.14) has a solution with a more complex structure than the Debye equation (2.16), since the operators

$$\frac{\partial}{\partial x} + \xi \frac{db_0}{dx}\frac{\partial}{\partial \zeta}, \quad \alpha_i\beta_e - \alpha_e\beta_i$$

are noncommutative.

REFERENCES

1. H. Hurwitz, R. Kilb, and G. Sutton, "The influence of tensor conductivity on current distribution in a magnetohydrodynamic generator," collection: The MHD Method of Energy Conversion [in Russian], 1963.

2. A. I. Morozov and A. P. Shubin, "Plasma flow between electrodes that have poor longitudinal conductivity," Teplofiz. vys. temperatur, 3, no. 6, 1965.

3. G. A. Lyubimov, "The viscous boundary layer on an electrode when the medium has variable electrical conductivity," PMM, 28, no. 5, 1964.

4. A. I. Morozov and L. S. Solov'ev, "On one similarity parameter in the theory of plasma flows," Dokl. AN SSSR, 164, no. 1, 1965.

5. A. I. Morozov and L. S. Solov'ev, "Two-dimensional flow of an ideally conducting compressible fluid with allowance for the Hall effect," Zh. tekhn. fiz., 34, no. 7, 1964.

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